Homework #2 (100 points) - Show all work on the following problems:

(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

**Problem 1 (20 points):** Check the divergence theorem  $\iiint (\nabla \cdot \vec{A}) dV = \oiint \vec{A} \cdot \vec{da}$  for the function  $\vec{A}(r,\theta,\phi) = r^2\hat{r}$ , using as your volume a sphere of radius R centered at the origin.

**Problem 2 (20 points):** Evaluate the following volume integrals.

a.  $\iiint (r^2 + \vec{r} \cdot \vec{a} + a^2) \delta^3(\vec{r} - \vec{a}) d\tau$  over all space  $(\vec{a} \text{ is a fixed vector of magnitude } a)$ 

b.  $\iiint (r^4 + r^2\vec{r} \cdot \vec{c} + c^4)\delta^3(\vec{r} - \vec{c})d\tau$  over a spherical volume with radius 6 centered at the origin, for the vector  $\vec{c} = 5\hat{x} + 3\hat{y} + 2\hat{z}$ .

**Problem 3 (20 points):** Take the vector functions  $\overrightarrow{F_1} = x^2 \hat{z}$  and  $\overrightarrow{F_2} = x \hat{x} + y \hat{y} + z \hat{z}$ .

- a. Calculate the divergence and curl of each one of these functions.
- b. Which one can be written as the gradient of a scalar function? For this one, find an example of a scalar function that has the right gradient.
- c. Which one can be written as the curl of a vector function? For this one, find an example of a vector function that has the right curl.

**Problem 4 (20 points):** Find the vector electric field a distance z above the center of a circular loop of radius R that carries a uniform line charge density  $\lambda$ .

**Problem 5 (20 points):** Find the vector electric field a distance z above the center of a flat circular disk of radius R that carries a uniform surface charge density  $\sigma$ .